Group Report 1964-9

Diplexer Using Side-Wall Couplers in One-Half Height Large X-Guide

J. A. Kostriza

17 January 1964

Prepared under Electronic Systems Division Contract AF 19 (628)-500 by

Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Lexington, Massachusetts

The work reported in this document was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology, with the support of the U.S. Air Force under Contract AF 19(628)-500.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

DIPLEXER USING SIDE-WALL COUPLERS IN ONE-HALF HEIGHT LARGE X-GUIDE

J. A. KOSTRIZA

Group 61

GROUP REPORT 1964-9

17 JANUARY 1964

LEXINGTON MASSACHUSETTS

ABSTRACT

A diplexer in one-half height large X-guide uses side-wall couplers. The analysis is based on the scattering matrix approach.

A compact unit is made possible because of an abrupt 180° E-plane bend.

This technical documentary report is approved for distribution.

Franklin C. Hudson

Franklin C. Hudson, Deputy Chief
Air Force Lincoln Laboratory Office

DIPLEXER USING SIDE-WALL COUPLERS IN ONE-HALF HEIGHT LARGE X-GUIDE

I. SCATTERING MATRIX OF TWO HYBRIDS IN CASCADE

A schematic of a side-wall coupler is shown in Fig. 1.

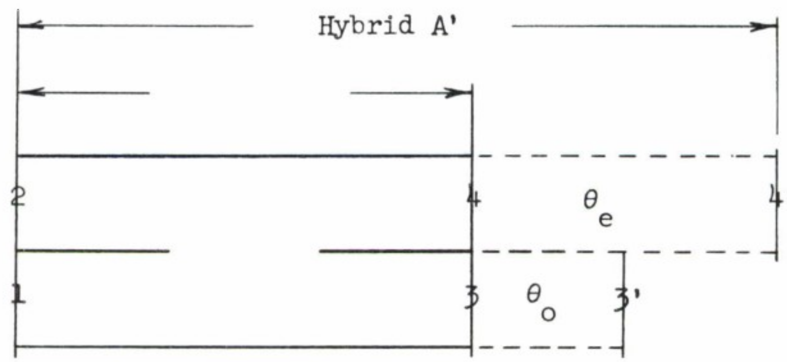

Fig. 1. Side-Wall Coupler Schematic, showing location of reference planes.

The reference planes are labeled 1, 2, 3 and 4. The scattering matrix is given by:

$$S = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & j \\ 0 & 0 & j & 1 \\ 1 & j & 0 & 0 \\ j & 1 & 0 & 0 \end{bmatrix} \qquad (1)$$

Now terminal 3 is moved to the right, through θ_o, to 3' and terminal 4 is moved through θ_e, to 4'. The scattering matrix for terminals 1, 2, 3', 4' becomes:

$$S' = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & e^{-j\theta_o} & je^{-j\theta_e} \\ 0 & 0 & je^{-j\theta_o} & e^{-j\theta_e} \\ e^{-j\theta_o} & je^{-j\theta_o} & 0 & 0 \\ je^{-j\theta_e} & e^{-j\theta_e} & 0 & 0 \end{bmatrix} \tag{2}$$

Equation (1) holds for hybrid A, whereas Eq. (2) holds for hybrid A'. If the output of A' is joined to a hybrid B whose scattering matrix is the same as that of A, a new four-port device results with terminal planes 1, 2, 3, 4 as shown in Fig. 2.

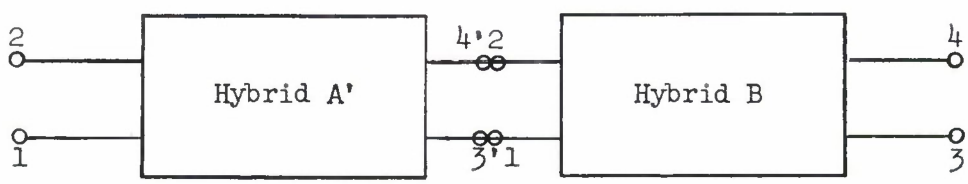

Fig. 2. Four - Port 1, 2, 3, 4 consisting of hybrid A' in cascade with hybrid B.

To join ports 4' with 2, and 3' with 1, the scattering equation b = Sa (b = reflected wave amplitudes, a = incident wave amplitudes) is written for both hybrids:

$$\left.\begin{aligned}
\sqrt{2}\, b_1 &= e^{-j\theta_o} a_{3'} + je^{-\theta_e} a_{4'} \\
\sqrt{2}\, b_2 &= je^{-j\theta_o} a_{3'} + je^{-j\theta_e} a_{4'} \\
\sqrt{2}\, b_{3'} &= e^{-j\theta_o} a_1 + je^{-j\theta_o} a_2 \\
\sqrt{2}\, b_{4'} &= je^{-j\theta_e} a_1 + e^{-j\theta_e} a_2
\end{aligned}\right\} \text{Hybrid A'}, \qquad (3)$$

$$\left.\begin{aligned}
\sqrt{2}\, B_1 &= A_3 + jA_4 \\
\sqrt{2}\, B_2 &= jA_3 + A_4 \\
\sqrt{2}\, B_3 &= A_1 + jA_2 \\
\sqrt{2}\, B_4 &= jA_1 + A_2
\end{aligned}\right\} \text{Hybrid B.} \qquad (4)$$

To "connect" Eqs. (3) and (4), the wave reflected from port 4' of hybrid A' must equal the wave incident on port 2 of hybrid B, etc., so that the following must hold:

$$\begin{aligned}
b_4' &= A_2 \quad \text{and} \quad b_3' = A_1 \\
a_4' &= B_2 \qquad\qquad a_3' = B_1
\end{aligned} \qquad (5)$$

Using Eqs. (5), (3) and (4), the composite structure of Fig. 2 may be characterized by Eq. (6) where small case letters are used throughout for reflected and incident wave amplitudes.

$$b_1 = \frac{a_3}{2}\left[e^{-j\theta_o} - e^{-j\theta_e}\right] + j\,\frac{a_4}{2}\left[e^{-j\theta_o} + e^{-j\theta_e}\right],$$

$$b_2 = j\,\frac{a_3}{2}\left[e^{-j\theta_o} + e^{-j\theta_e}\right] + \frac{a_4}{2}\left[-e^{-j\theta_o} + e^{-j\theta_e}\right],$$

$$b_3 = \frac{a_1}{2}\left[e^{-j\theta_o} - e^{-j\theta_e}\right] + j\,\frac{a_2}{2}\left[e^{-j\theta_o} + e^{-j\theta_e}\right], \qquad (6)$$

$$b_4 = j\,\frac{a_1}{2}\left[e^{-j\theta_o} + e^{-j\theta_e}\right] + \frac{a_2}{2}\left[-e^{-j\theta_o} + e^{-j\theta_e}\right].$$

II. DIPLEXER REQUIREMENTS

To achieve diplexer action, the following conditions must be met:

1. A signal of frequency f_1, incident at port 1, emerges out of port 3 (or port 4) with negligible coupling to port 4 (or port 3), and
2. A signal of frequency f_2, incident at port 2, emerges out of port 3 (or port 4) with negligible coupling to port 4 (or port 3), and
3. Zero or small coupling between ports 1 and 2.

If the signal f_1 is applied at port 1 (i.e., $a_2 = 0$), then:

$$b_3\ (\text{at } f_1) = \frac{a_1}{2}\left[e^{-j\theta_o} - e^{-j\theta_e}\right],$$

$$b_4\ (\text{at } f_1) = j\,\frac{a_1}{2}\left[e^{-j\theta_o} + e^{-j\theta_e}\right]. \qquad (7)$$

From the above:

when $\theta_o - \theta_e = \pm\, 2\pi n$ ($n = 0, 1, 2$---), then $b_3 = 0$, and (8)

when $\theta_o - \theta_e = \pm\, 2\pi\left[m + \frac{1}{2}\right]$ ($m = 0, 1, 2$---), then $b_4 = 0$. (9)

If the signal f_2 is applied at port 2 (i.e., $a_1 = 0$), then:

$$b_3 \text{ (at } f_2) = j \frac{a_2}{2} \left[e^{-j\theta_o} + e^{-j\theta_e} \right],$$

$$b_4 \text{ (at } f_2) = \frac{a_2}{2} \left[- e^{-j\theta_o} + e^{-j\theta_e} \right]. \tag{10}$$

From Eq. (10) it follows that:

$$\text{when } \theta_o - \theta_e = \pm 2\pi \left[m + \frac{1}{2} \right] (m = 0, 1, 2\text{---}), \text{ then } b_3 = 0, \text{ and} \tag{11}$$

$$\text{when } \theta_o - \theta_e = \pm 2\pi n \; (n = 0, 1, 2\text{---}), \text{ then } b_4 = 0. \tag{12}$$

III. COMMON OUTPUT IS PORT 4

Assume that b_4 is of interest at both frequencies f_1 and f_2. Then Eqs. (8) and (11) must be satisfied.

$$Pb_3 \, (f_1) = \frac{1}{2} b_3 \, b_3{}^* = \frac{1}{4} (1 - \cos\theta_1), \text{ where } \theta_1 = \theta_o - \theta_e,$$

$b_3{}^* = b_3$ conjugate, and Pb_3 = power reflected at port 3.

$$Pb_4 \, (f_1) = \frac{1}{4} (1 + \cos\theta_1),$$

$$P_{in} \, 1 \, (f_1) = \frac{1}{2} a_1 \, a_1{}^* = \frac{1}{2}; \; Pb_3 + Pb_4 = \frac{1}{2}.$$

$$\text{I.L. } (f_1) = 10 \log \frac{P_{in} \, 1}{Pb_4} = 10 \log \frac{2}{1 + \cos\theta_1}.$$ [1]

If $\theta_1 = 2\pi n \pm \delta_1$, $\cos\theta_1 = \cos\delta_1$, and

1. L. J. Ricardi, "A Diplexer Using Hybrid Junctions," Technical Report No. 255 (U), Lincoln Laboratory, M.I.T. (7 February 1962).

$$\text{I.L. } (f_1) = 10 \log \frac{2}{1 + \cos\delta_1}. \tag{13}$$

$$Pb_3\ (f_2) = \frac{1}{4}\,(1 + \cos\theta_2), \text{ where } \theta_2 = \theta_o - \theta_e,$$

$$Pb_4\ (f_2) = \frac{1}{4}\,(1 - \cos\theta_2),$$

$$P_{in}\,②\,(f_2) = \frac{1}{2}\, a_2 \cdot a_2{}^* = \frac{1}{2};\ Pb_3 + Pb_4 = \frac{1}{2}.$$

$$\text{I.L. } (f_2) = 10 \log \frac{P_{in}\,②}{Pb_4} = 10 \log \frac{2}{1 - \cos\theta_2}.$$

If $\theta_2 = 2\pi m + \pi \pm \delta_2$, $\cos\theta_2 = -\cos\delta_2$, and

$$\text{I.L. } (f_2) = 10 \log \frac{2}{1 + \cos\delta_2}. \tag{14}$$

<u>Input mismatch at f_1, f_2.</u>

At f_1 : $a_2 = 0$, $a_3 = \Gamma_3 b_3$, $a_4 = \Gamma_4 b_4$ where Γ is the voltage reflection factor,

From Eq. (6):

$$b_1 = \frac{a_1}{4}\left[\Gamma_3 \left(e^{-j\theta_o} - e^{-j\theta_e}\right)^2 - \Gamma_4 \left(e^{-j\theta_o} + e^{-j\theta_e}\right)^2\right],$$

$$\Gamma_{in\,①} = \frac{b_1}{a_1} = \frac{e^{-j2\theta_o}}{4}\left[\Gamma_3\,(1 - e^{j\theta})^2 - \Gamma_4\,(1 + e^{j\theta})^2\right],$$

with $\theta = \theta_o - \theta_e$.

But $\theta_1 = 2\pi n + \delta_1$, $e^{j\theta_1} = e^{j\delta_1}$,

$$\therefore \Gamma_{in}(1) = \frac{e^{-j2\theta_o}}{4}\left[\Gamma_3\left(1 - e^{j\delta_1}\right)^2 - \Gamma_4\left(1 + e^{j\delta_1}\right)^2\right].$$

At f_2: $a_1 = 0$, $a_3 = \Gamma_3 b_3$, $a_4 = \Gamma_4 b_4$.

From Eq. (6):

$$b_2 = \frac{a_2}{4}\left[-\Gamma_3\, e^{-j2\theta_o}\left(1 + e^{j\theta}\right)^2 + \Gamma_4\, e^{-j2\theta_o}\left(-1 + e^{j\theta}\right)^2\right],$$

with $\theta = \theta_o - \theta_e$:

$$\Gamma_{in}(2) = \frac{b_2}{a_2} = \frac{e^{-j2\theta_o}}{4}\left[-\Gamma_3\left(1 + e^{j\theta}\right)^2 + \Gamma_4\left(-1 + e^{j\theta}\right)^2\right].$$

But $\theta_2 = 2\pi m + \pi + \delta_2$, $e^{j\theta_2} = -e^{j\delta_2}$,

$$\therefore \Gamma_{in}(2) = \frac{e^{-j2\theta_o}}{4}\left[-\Gamma_3\left(1 - e^{j\delta_2}\right)^2 + \Gamma_4\left(-1 - e^{j\delta_2}\right)^2\right].$$

IV. COMMON OUTPUT IS PORT 3

At f_1, from Eq. (9), $\theta_1 = \theta_o - \theta_e = 2\pi\left[m + \frac{1}{2}\right] \pm \delta_1$, $\cos\theta_1 = -\cos\delta_1$.

$$\text{I.L. }(f_1) = 10 \log \frac{P_{in}(1)}{Pb_3} = 10 \log \frac{2}{1 - \cos\theta_1} = 10 \log \frac{2}{1 + \cos\delta_1}. \qquad (15)$$

At f_2, from Eq. (12), $\theta_2 = \theta_o - \theta_e = 2\pi n \pm \delta_2$, $\cos\theta_2 = \cos\delta_2$.

$$\text{I.L. } (f_2) = 10 \log \frac{P_{in\,2}}{Pb_3} = 10 \log \frac{2}{1 - \cos\theta_2} = 10 \log \frac{2}{1 + \cos\delta_2}. \qquad (16)$$

V. SAMPLE DESIGNS

A. Design for $f_1 = 7.75$ Kmc, $f_2 = 8.35$ Kmc, power out of port 4.

$$\text{Set } \theta_1 = \frac{L}{\lambda g_1} \cdot 2\pi = 2\pi n \text{ } \underline{\text{or}} \text{ } L = n\lambda g_1.$$

$$\text{Set } \theta_2 = \frac{L}{\lambda g_2} \cdot 2\pi = 2\pi \left(m + \frac{1}{2} \right) + \delta_2.$$

In large X-guide, the minimum L comes out 8.28", with I.L. $(f_1) = 0$ and I.L. $(f_2) = .12$ db.

B. Design for $f_1 = 7.75$ Kmc, $f_2 = 8.35$ Kmc, power out of port 3.

$$\text{Set } L = \left(m + \frac{1}{2} \right) \lambda g_1.$$

$$\therefore \theta_2 = \theta_o - \theta_e = \frac{L}{\lambda g_2} . 2\pi = 2\pi n + \delta_2.$$

In large X-guide, the minimum L is 7.245", with I.L. $(f_1) = 0$ db, I.L. $(f_2) = .02$ db.

VI. DIPLEXER IN $\frac{1}{2}$ HEIGHT LARGE X-GUIDE

For satellite applications, to limit weight, it was decided to fabricate the L = 7.245" design. Also, the large X-guide was decreased to $\frac{1}{2}$ height. An MDL large X-guide side-wall coupler was decreased to half height and the capacitive dimple was replaced by a #4-40 screw. With a screw penetration of approximately 0.120", P_{13} and P_{14} were within 0.1 db at 7750, P_{12} was greater than 30 db, and VSWR was 1.05. At 8350, the corresponding values were 0.1 db, 23 db and 1.12 VSWR.

A second hybrid gave somewhat worse results:

7750 —— 0.1 db, > 30 db, 1.05 VSWR;

8350 —— 0.4 db, 20 db and 1.09 VSWR.

The screw sensitivity was about 0.5 db/turn w/r P_{13} and P_{14}, with a measurable but small effect on input VSWR's.

In Fig. 2, imagine that hybrid B, being pivoted at ports 1, 2 is lifted upward through 180° and then is slid over until it lies exactly over hybrid A. Ports 4' and 2, and 3' and 1 are now connected by an abrupt 180° bend as illustrated in Fig. 3.

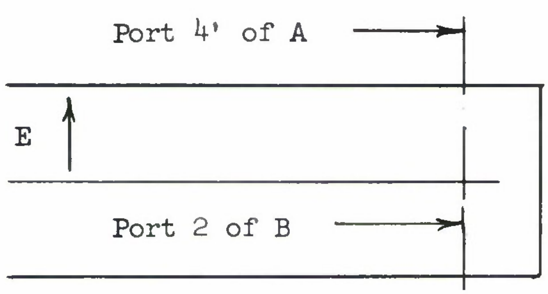

Fig. 3: 180° E-Plane Bend in $\frac{1}{2}$ Height Large X-Guide.

The VSWR characteristics of the bend in Fig. 3 are:

7750 - 1.05

8050 - 1.02

8350 - 1.02

The complete diplexer appears in Figs. 4 and 5. Because of soldering difficulties, it was not possible to align each $\frac{1}{2}$ height hybrid for optimum behavior prior to joining. Therefore, each hybrid has a tuning screw for power-split trimming (where the dimple had been) and a "shorting" screw in the line

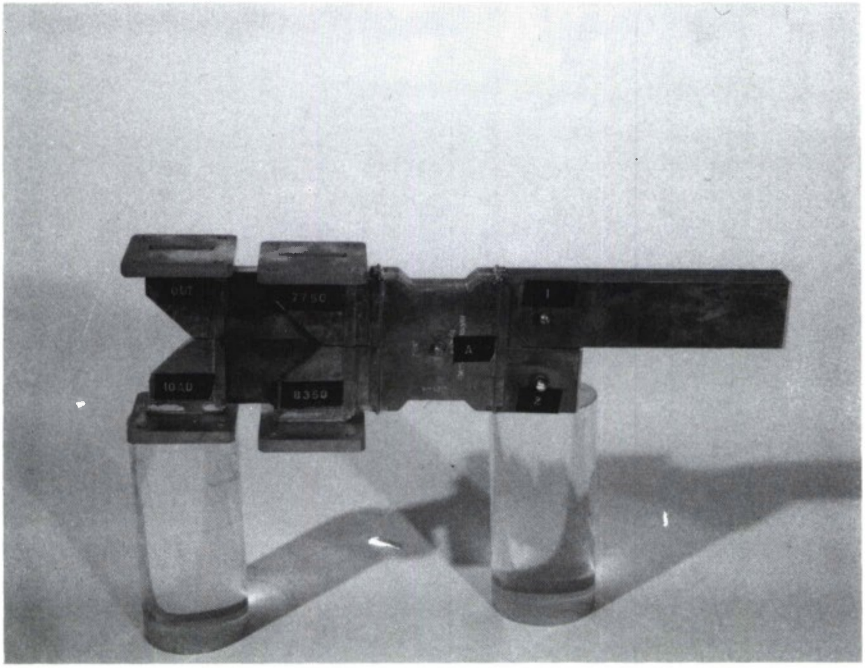

Fig. 4: Photograph of Completed Diplexer

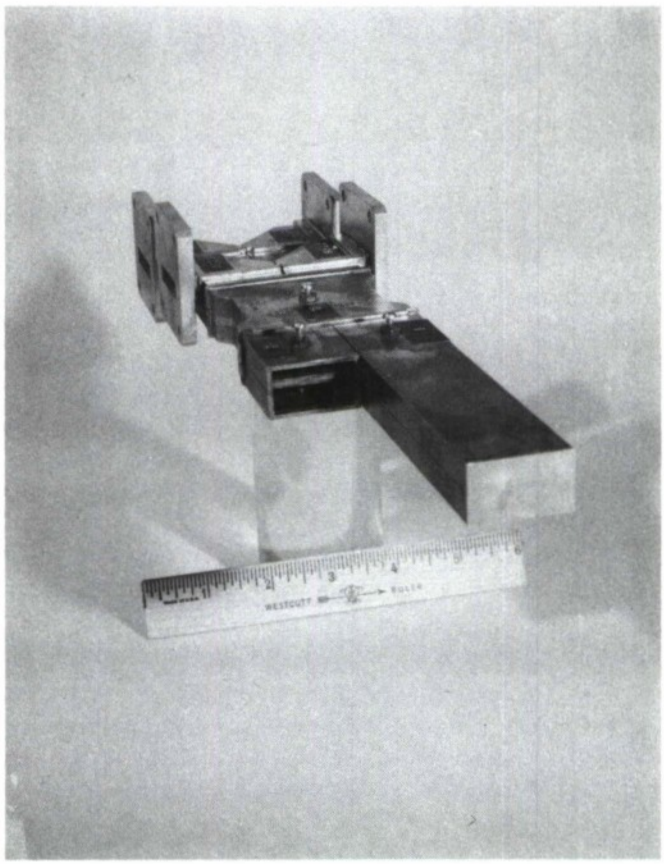

Fig. 5: Photograph of Diplexer
with Inside View of 180° Bend

past ports 3 and 4. It was hoped that with the shorting screw in position, it would be possible to set the power-split screw for minimum VSWR and so balance each hybrid.

Circuit A was found to be somewhat worse than that experienced on the two preliminary $\frac{1}{2}$ height hybrids, giving VSWR's of 1.12 at 7750 and 1.21 at 8350. Circuit B gave VSWR's of 1.38. The reason for such inferior performance of B is not known.

In view of the poor performance of hybrid B, all four shorting screws were used as tuning elements, in addition to the two power-split screws. By a converging process, the diplexer was tuned to the following characteristics:

<u>7750</u>	<u>8350</u>
I.L. = 0.3 db (= P_{13})	I.L. = 0.5 db (= P_{23})
P_{12} = 32 db	P_{21} = 27 db
P_{14} = 30 db	P_{24} = 22 db
VSWR 1 $\approx$ 1.3	VSWR 2 = 1.28

DISTRIBUTION LIST

Division 6

T. F. Rogers
G. P. Dinneen

Group 61

L. J. Ricardi (10)
R. N. Assaly
W. C. Danforth
M. E. Devane
J. Kostriza
B. F. LaPage
C. A. Lindberg
J. B. Rankin
A. Sotiropoulos
Leon Niro
R. J. Pieculewicz

Group 62

P. Rosen (2)

Group 63

W. E. Morrow (2)

Group 64

P. E. Green

Group 65

H. Sherman

Group 66

R. T. Prosser